

# Engineering massive quantum memories by topologically time-modulated spin rings

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**Abstract.** We introduce a general scheme to realize perfect storage of quantum information in systems of interacting qubits. This novel approach is based on *global* external controls of the Hamiltonian, that yield time-periodic inversions in the dynamical evolution, allowing a perfect periodic quantum state reconstruction. We illustrate the method in the particularly interesting and simple case of spin systems affected by  $XY$  residual interactions with or without static imperfections. The global control is achieved by step time-inversions of an overall topological phase of the Aharonov-Bohm type. Such a scheme holds both at finite size and in the thermodynamic limit, thus enabling the massive storage of arbitrarily large numbers of local states, and is stable against several realistic sources of noise and imperfections.

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## 1. Introduction

In the attempt to pave the way to the realization of scalable schemes for quantum computation, much theoretical work has been recently aimed at developing suitable strategies for the efficient processing and the coherent transfer of quantum information [1, 2, 3, 4, 5]. Besides these two fundamental aspects, a further crucial requirement for the realization of scalable quantum computers is the possibility to store quantum data on time scales at least comparable to those needed for the computational process. In particular, it is very important to introduce systems acting as stable and robust quantum memories that recover and conserve large sets of quantum states that would be otherwise usually lost in very short times, due to quantum diffusion and decoherence [5].

To ensure stable information storage in a quantum register, many different noise-evading schemes have been proposed [5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. All these works can be roughly classified in two different groups: the first includes schemes based on some error correction technique. The remaining proposals exploit some intrinsic property of the quantum register that leaves some specific subsets of quantum states unaffected along the temporal evolution (decoherence-free subspace schemes). These latter approaches provide, in principle, the complete solution to the problem of quantum information storage, but, unfortunately, they are extremely sensitive to almost any source of imperfection. On the contrary, the schemes based on quantum error correction techniques are characterized by a dynamics that allows at any time the unambiguous reconstruction of the initial information. The main trouble with error correction techniques lies in the fact that usually only very few states can be effectively accessed to store information and, hence, relatively large arrays of qubits are needed to memorize relatively small amounts of information.

In the present paper we introduce a new approach to quantum state storage based on the idea of time-controlled periodic dynamics that allows a perfect, periodic reconstruction of a generic initial state. To demonstrate and describe it, let us first recall that an ideal quantum register can be considered as a set of isolated identical qubits subject to a local Hamiltonian

$$\hat{H}_0 = B \sum_i \sigma_i^z, \quad (1)$$

where  $B$  is the half gap between the two energy levels of each spin[15]. However, in realistic situations, the register is subject to noise caused by disorder in the local gap and by interactions both with the substrate environment and between the qubits. Typically, at least at sufficiently low temperatures or sufficiently weak coupling with the background substrate, the interaction with the external environment takes place on time scales much slower than those associated to the computational process. For instance, in the case of quantum gate operations with hyperfine levels in trapped ions, the ratio  $t_{\text{gate}}/t_{\text{decoh}}$  can be as small as  $10^{-9}$  [16, 17]. Then, the corrupting effects that take place on the same time scales of the computational processes, and thus need to be addressed first, are those due to the unavoidable presence of the residual, deterministic and/or random inter-qubit interactions [16, 17]. These interactions cause, in general, fast quantum diffusion and, as a consequence, the complete corruption of the information one wishes to store and process. Therefore, even before considering the effects of thermal and environmental decoherence, realistic quantum registers in the presence of irreducible noise and imperfections must be

described by Hamiltonians of the form [15, 18]:

$$\hat{H}_{tot} = \hat{H}_0 + \hat{H}_{err}, \quad (2)$$

where  $\hat{H}_{err}$  is the residual inter-qubit Hamiltonian. In several physical situations, it can be described by  $XY$  interaction terms:

$$\hat{H}_{err} = - \sum_i (\lambda + \eta_i) \left( \sigma_i^+ \sigma_{i+1}^- + H.c. \right), \quad (3)$$

where the random variables of vanishing mean  $\eta_i$  are the local imperfections in the global, averaged nearest-neighbor coupling amplitude  $\lambda$ , and  $\sigma_k^\pm = \sigma_k^x \pm i\sigma_k^y$ . Such a site-dependent  $XY$  model applies immediately to spin-1/2 based quantum registers (such as in NMR devices), but gives as well an effective description of a register based on hopping and/or interacting particles on a lattice, in the presence of an energy gap such that only two local states on each lattice site can be considered [19], so that the material particles are mapped in spin-1/2 systems. The storing scheme that we shall describe in the present work holds in general for many classes of inter-qubit interaction Hamiltonians, for instance  $XXZ$  interactions, but in a more limited range of validity when dealing with imperfections [20]. Therefore, in the following, we will restrict our analysis to physical systems and regimes such that the residual inter-qubit couplings can be well described by  $XY$  interaction Hamiltonians, including local imperfections.

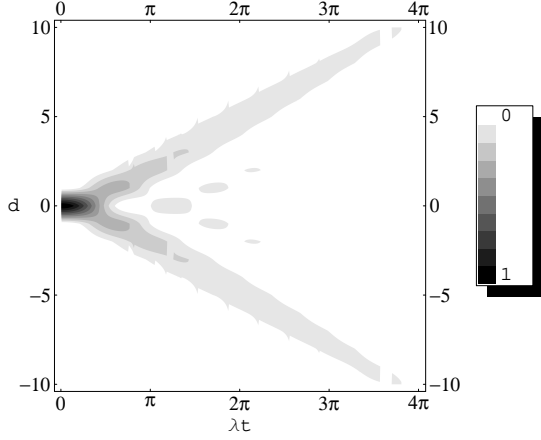
As already mentioned, the presence of  $\hat{H}_{err}$ , even if  $\lambda \ll B$  and  $\eta_i = 0 \forall i$ , rapidly destroys the storing capacity of the register: immediately after having stored an *initial state*  $|\psi(t=0)\rangle \equiv |\chi\rangle$ , such state starts to evolve and diffuse indefinitely. To illustrate and measure this effect, we can follow the behaviour of the time-dependent two-dimensional overlap  $\mathcal{F}_d(t) \equiv |\langle \chi | \psi(t) \rangle|^2$  for different stored initial states and physical situations [21]. Let us then consider a closed (periodic boundary conditions), unmodulated chain with  $XY$  nearest neighbour residual interactions between the spins, and let us introduce the set of one-magnon, locally excited states

$$|\Psi_d\rangle = |\uparrow_d\rangle \prod_{i \neq d} |\downarrow_i\rangle, \quad (4)$$

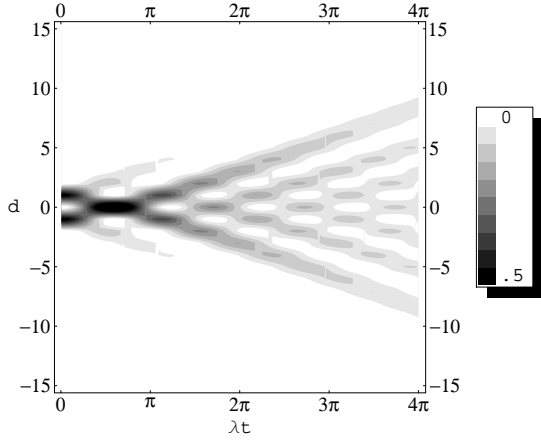
where the product involves all the qubits of the register except the one at site  $d$ . In Fig. 1 we take the initial state  $|\chi\rangle$  to be  $|\Psi_0\rangle$ , i.e. the one-magnon excitation placed on site  $d = 0$ , while in Fig. 2 the initial state is taken to be the linear combination with equal weights of one-magnon excitations placed on sites  $d = 1$  and  $d = -1$ . By looking both at Fig. (1) and Fig. (2), we see that, irrespective of the different set of physical parameter and stored states one considers, the quantum state diffusion grows indefinitely in time and the initial state is never recovered at any site of the lattice. Consequently, the one-dimensional fidelity of the initially stored state at a fixed position  $d$  in the lattice quickly decrease, undergoing partial revivals, as shown in Fig. (3). It is worth noticing that in a non modulated register the fidelity of an initial superposition is more robust against quantum diffusion. This intrinsic property is helpful in the case of modulated (controlled) registers.

## 2. Storing quantum information by topologically modulated spin chains and time-inverted quantum dynamics

To overcome this major obstacle in the construction of *prima facie* working models of a quantum register we will introduce a *global* (that is, site-independent) control of the

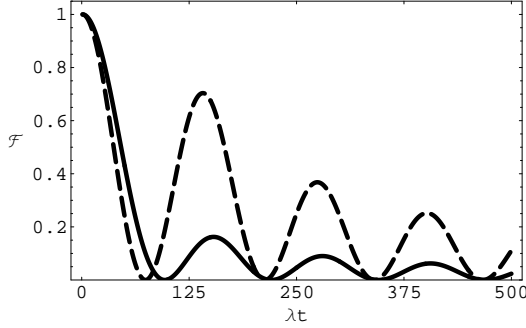


**Figure 1.** Unmodulated register: contour plot showing the evolution of the bidimensional overlap  $\mathcal{F}_d(t)$ , starting from the initial state  $|\Psi_0\rangle$ , as a function of the distance  $d$  from site 0 ( $y$ -axis) and of the dimensionless time  $\lambda t$  in atomic units  $\hbar = 1$  ( $x$ -axis). Here  $B = 100\lambda$ . The value of the overlap increase from white ( $\mathcal{F}_d(t) = 0$ ) to black ( $\mathcal{F}_d(t) = 1$ ).



**Figure 2.** Unmodulated register: contour plot showing the evolution of the two-dimensional overlap  $\mathcal{F}_d(t)$ , for the initial state  $|\chi\rangle = (|\Psi_1\rangle + |\Psi_{-1}\rangle)/\sqrt{2}$ , as a function of the distance  $d$  from site 0 ( $y$ -axis) and of the dimensionless time  $\lambda t$  ( $x$ -axis) in atomic unit  $\hbar = 1$ . The value of the overlap increases from white ( $\mathcal{F}_d(t) = 0$ ) to black ( $\mathcal{F}_d(t) = 0.5$ ).

total Hamiltonian  $\hat{H}_{tot}$  able to force a periodic time reconstruction of the initial state. In the presence of a generic time-dependent Hamiltonian, evolution of a quantum state can be determined in the most general terms by resorting to the Dyson series representation. Obviously, the structural complexity of the Dyson evolution integral does not allow to identify all the possible Hamiltonian dynamical evolutions yielding perfect, time-periodic quantum state reconstruction. However, the Dyson series can



**Figure 3.** Unmodulated register: One-dimensional fidelity  $\mathcal{F}_0(t)$  at fixed lattice site  $d = 0$ , as a function of the dimensionless time  $\lambda t$  for different initially stored states. The solid line represents the evolution of the fidelity with initial state taken to be the single-magnon excitation  $|\Psi_0\rangle$  localized at site  $d = 0$ . The dashed line represents the evolution of the one-dimensional fidelity for the initial superposition state  $|\chi\rangle = (|\Psi_1\rangle + |\Psi_{-1}\rangle)/\sqrt{2}$ . Notice that superpositions are more robust against quantum diffusion than non-superpositional, localized excitations.

be easily resummed when the Hamiltonian enjoys the property to self-commute at different times:  $[\hat{H}(t), \hat{H}(t')] = 0$  for all  $t$  and  $t' \neq t$ . Thus, our first prescription concerns a quantum register described by a time-periodic modulated Hamiltonian, commuting at all times, either when applied to all possible states, or for a subspace of the whole Hilbert space of states.

Under the time-commutation hypothesis, we can determine a complete set of states  $\{|\alpha\rangle\}$  that are eigenstates of the time-dependent Hamiltonian at all times even if the corresponding energy eigenvalues are time-dependent functions  $\varepsilon^\alpha(t)$ . Because the energy eigenstates form a complete basis set, the initial state  $|\chi\rangle$  can be written as a linear combination:  $|\chi\rangle = \sum_\alpha c_\alpha |\alpha\rangle$ , with  $c_\alpha = \langle\alpha|\chi\rangle$ , and the sum runs over the complete set of eigenstates. It is then simple to write the evolution of the initial state after a time  $t > 0$ ,

$$|\psi(t)\rangle = \sum_\alpha c_\alpha \exp\left(-i \int_0^t \varepsilon^\alpha(\tau) d\tau\right) |\alpha\rangle. \quad (5)$$

From Eq. (5) it is immediate to see that if at a certain time  $T > 0$  all the integrals in the sum are equal or differ from each other by integer multiples of  $2\pi$ , then the initial state  $|\chi\rangle$  is perfectly reconstructed, but for an irrelevant global phase factor. This is then the second requirement that one needs to impose on the time-modulated dynamics in order to realize perfect time-periodic quantum state storage.

These two basic requirements can be implemented successfully for spin systems with XY interactions by a scheme of quantum control based on time-inverted dynamical evolutions realized by suitably engineered time-dependent global phase factors. We then consider the following situation. In the presence of a simple geometry realized by placing a tiny solenoid in the center of a circular ring (periodic boundary conditions) of spins sitting at regularly spaced lattice sites, all the nearest-neighbor interaction amplitudes become complex by acquiring the same site-independent, global phase  $\theta$  proportional to the magnetic flux  $\phi$  linked to the ring:  $\theta \propto \phi/N$ , where  $N$  is the total number of qubits in the ring. The solenoid field can then be modulated in time to achieve the control needed for perfect quantum state storage. In the presence

of a time-variable linked flux, the total Hamiltonian Eq. (2) becomes time-dependent and reads:

$$\hat{H}_{tot}(t) = - \sum_i (\lambda + \eta_i) \left( e^{i\theta(t)} \sigma_i^+ \sigma_{i+1}^- + H.c. \right) + B \sum_i \sigma_i^z. \quad (6)$$

Incidentally, we note that this scheme can apply as well to more general systems of hopping material particles, thanks to the “Peierls” effect, i.e. the fact that if charged particles are in the presence of a linked magnetic flux, the real-valued hopping amplitude between particles (in spin language, the nearest-neighbor coupling), is transformed in a complex-valued quantity [22].

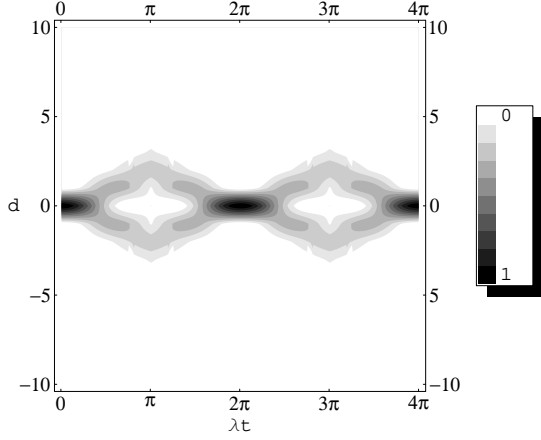
Clearly, not all time modulations of the phase factor can realize the desired perfect state storage. The first constraint to be imposed is commutativity at different times:  $[\hat{H}_{tot}(t), \hat{H}_{tot}(t')] = 0$ . This property is verified if and only if  $\theta(t) - \theta(t') = k\pi$ , with  $k$  integer. This implies that during the entire time evolution the phase must be modulated in regular periodic jumps (steps) between two constant values  $\theta_0$  and  $\theta_0 + \pi$  (step-phase modulation).

Concerning the property that at a certain given time  $T$ , all the time integrals appearing in the expression Eq. (5) must be equal (or differ by a trivial phase factor integer multiple of  $2\pi$ ), let us observe that, independently of the values of  $\theta$ , the local term  $\hat{H}_0$  (ideal register) commutes with the  $XY$  residual interaction terms in the time-dependent Hamiltonian Eq. (6). Hence, there exists a complete set of eigenstates of  $\hat{H}_c(t)$  that are as well simultaneous eigenstates of both the local and the interaction terms. Then, for all eigenvalues  $\varepsilon^\alpha(\theta_0)$  associated to the eigenstates  $|\alpha\rangle$ , we have that:  $\varepsilon^\alpha(\theta_0) = \varepsilon_c^\alpha(\theta_0) + \varepsilon_l^\alpha$ , where  $\varepsilon_c^\alpha(\theta_0)$  and  $\varepsilon_l^\alpha$  are, respectively, the interaction and the local contributions to the energy. On the other hand, when  $\theta$  passes from the value  $\theta_0$  to  $\theta_0 + \pi$ , the coupling contribution to the energy changes sign while the local one remains unchanged, so that  $\varepsilon^\alpha(\theta_0 + \pi) = -\varepsilon_c^\alpha(\theta) + \varepsilon_l^\alpha$ . Therefore, any two energy eigenvalues  $\varepsilon^\alpha(\theta)$  and  $\varepsilon^\alpha(\theta + \pi)$  corresponding to the same eigenstate  $|\alpha\rangle$ , differ only in the sign of the interaction component. Then, selecting a regular step time modulation of the phase of the form

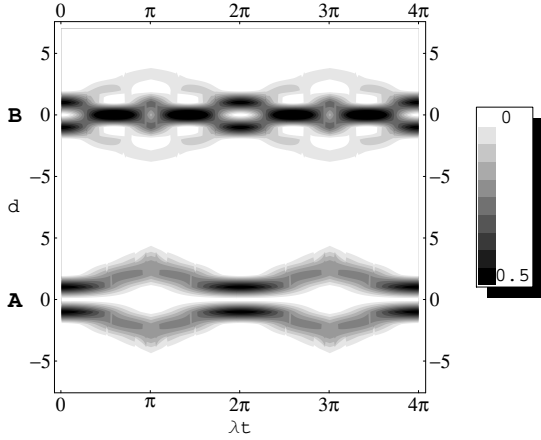
$$\theta(t) = \begin{cases} \theta, & 0 \leq t < T/2; \\ \theta + \pi, & T/2 \leq t < T, \end{cases} \quad (7)$$

periodically repeated for any  $t \geq T$ , we obtain that the contribution of the residual interaction Hamiltonian to the quantum state time evolution vanishes at any time  $t = mT$  with  $m$  arbitrary integer. Consequently, the effects of the undesired, residual interqubit  $XY$  interactions on the quantum register are completely eliminated, *even in the presence of local, static imperfections in the couplings*. Moreover, the result is valid for *any kind* of distribution of the noise on the couplings, be it Gaussian, or uniform, or any other probability distribution. The effects of the storing scheme are illustrated in Fig. (4), Fig. (5), and Fig. (6).

In Fig. (4) we again plot the time evolution of the two-dimensional overlap  $\mathcal{F}_d(t)$ , for the same initial state considered in Fig. (1), but now under the action of Hamiltonian  $\hat{H}_{tot}(t)$  Eq. (6). At striking variance with the unmodulated case reported in Fig. (1), the step-phase modulated register realizes exact, time-periodic coherent revivals of the initial state. Moreover, Fig. (4) shows that the overall spatial diffusion of the state is always confined in a well defined and extremely narrow region of the ring. Because the topological control remains exact in the thermodynamic limit, the number of stored states can be arbitrarily large, possibly macroscopic. Finally, the evolution of this kind of “non-superpositional” initial state is not affected by the

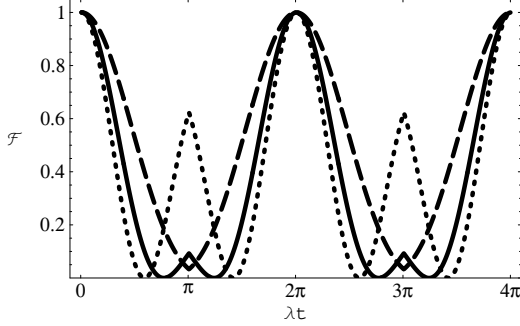


**Figure 4.** Step-periodic time-modulation of the phase, Eq. (7): contour plot showing the evolution of the bidimensional overlap  $\mathcal{F}_d(t)$ , for the same initial state considered in Fig. (1), as a function of the distance  $d$  from site 0 ( $y$ -axis) and of the dimensionless time  $\lambda t$  in atomic units  $\hbar = 1$  ( $x$ -axis). Here  $\lambda T = 2\pi$  and  $\theta = \pi/2$ . The value of the overlap increases from white ( $\mathcal{F}_d(t) = 0$ ) to black ( $\mathcal{F}_d(t) = 1$ ).



**Figure 5.** Step-periodic time-modulation of the phase, Eq. (7): contour plot showing the evolution of the two-dimensional overlap  $\mathcal{F}_d(t)$ , for the initial state  $|\chi\rangle = (|\Psi_1\rangle + |\Psi_{-1}\rangle)/\sqrt{2}$ , as a function of the distance  $d$  from site 0 ( $y$ -axis) and of the dimensionless time  $\lambda t$  ( $x$ -axis). Here  $\lambda T = 2\pi$ . In graph A),  $\theta$  flips periodically between  $-\pi/2$  and  $\pi/2$ . In graph B) it flips periodically between 0 and  $\pi$ . The value of the overlap increases from white ( $\mathcal{F}_d(t) = 0$ ) to black ( $\mathcal{F}_d(t) = 0.5$ ).

choice of the initial phase  $\theta$ . However, for the purposes of quantum computation, the most interesting and desirable scope is obviously the storage of superposition states.



**Figure 6.** Step-periodic time-modulation of the phase Eq. (7): One-dimensional fidelity  $\mathcal{F}_0(t)$  at fixed lattice site  $d = 0$  as a function of the dimensionless time  $\lambda t$  ( $x$ -axis) for different constant values  $\theta_0$  of the phase and different initially stored states in the case of  $\lambda T = 2\pi$ . The solid line represents the evolution of the fidelity when the initial state is the single-magnon excitation  $|\Psi_0\rangle$  and the phase is  $\theta_0 = \pi/2$ . The dotted and the dashed lines both represent the evolutions of the initial superposition states  $|\chi\rangle = (|\Psi_1\rangle + |\Psi_{-1}\rangle)/\sqrt{2}$ . The dotted line is for the case  $\theta_0 = 0$ ; the dashed line for  $\theta_0 = \pi/2$ . Notice that, according to the assigned value of the phase, the speed of revival gain for the superpositional state can be larger or smaller than that of the nonsuperpositional state.

Remarkably, if the initial state  $|\chi\rangle$  is a superposition of the form

$$|\chi\rangle = \sum_{i=1}^M a_i |\Psi_{d_i}\rangle,$$

i.e. such that the excitations are distributed among different sites, the step-phase control again yields perfect periodic state reconstruction with extremely limited intermediate spatial spread, although some of the details of the evolution at intermediate times can change significantly. As an example, in Fig. (5) we plot the overlap  $\mathcal{F}_d(t)$  driven by the Hamiltonian Eq. (6), starting from the initial superposition state  $|\chi\rangle = (|\Psi_1\rangle + |\Psi_{-1}\rangle)/\sqrt{2}$ , and for different values of the phase  $\theta$ . These results can be further generalized to any initial state  $|\chi\rangle$  with an arbitrary number of excitations or flipped spins (arbitrarily many magnons). In this case, one finds the same coherent time-periodic revival of the state as in the one-magnon case, while the spatial spread becomes a function of the size of the region along which the initial state is extended, but remains in any case finite and limited.

To better compare the different situations that can be realized using the step-modulated or the unmodulated phase, we show in Fig. (6) the behaviour of the one-dimensional fidelity at fixed lattice site  $d = 0$ , in the presence of periodic step modulation of the phase for different initial states and different constant values  $\theta_0$  of the phase. It is remarkable to notice that, depending on the different constant values of the phase that one chooses as reference values to operate with the control scheme, the approach to the full revival for the superpositional states can be larger or smaller than that of the non-superpositional states. An important point is to identify all the possible classes of states that can be perfectly stored using our method. Obviously, the answer to this question involves not only the interaction Hamiltonian, but the local, diagonal term  $\hat{H}_0$  as well. If the period  $T$  of the step-phase control is gauged so



that

$$BT = 2l\pi$$

with  $l$  integer, then *any initial quantum state* is reconstructed *exactly* at all times  $t$  integer multiples of  $T$ . If instead  $BT \neq 2l\pi$ , perfect quantum state storage is still achieved in the subspace of all states that are linear combinations of local states with equal magnetization. To gain further understanding of this result, let us consider generalizations of the locally excited states  $|\Psi_d\rangle$  by introducing the  $n$ -times locally excited states

$$|\Psi_{d_1, \dots, d_n}^{(n)}\rangle = \prod_{j=d_1, \dots, d_n} |\uparrow_j\rangle \prod_{i \neq d_1, \dots, d_n} |\downarrow_i\rangle. \quad (8)$$

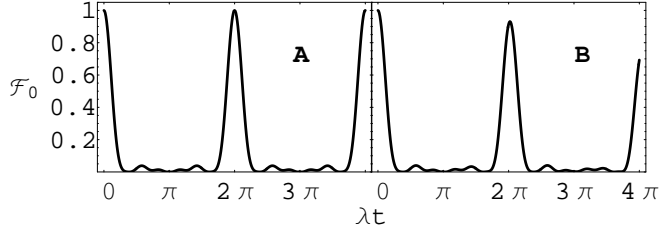
We thus wish to analyze the case in which the state to be stored is a superposition of local states with different values of the magnetization, and either integer or real values of the ratio  $B/\lambda$ . To this end, in Fig. (7) we plot the one-dimensional fidelity  $\mathcal{F}_0(t)$  at fixed lattice site  $d = 0$  for an initially stored state of the form

$$|\chi\rangle = -(\sqrt{2}/3)|\Psi_{20}\rangle + (\sqrt{1}/3)|\Psi_{72}\rangle + (\sqrt{2}/\sqrt{3})|\Psi_{0,5}^{(2)}\rangle,$$

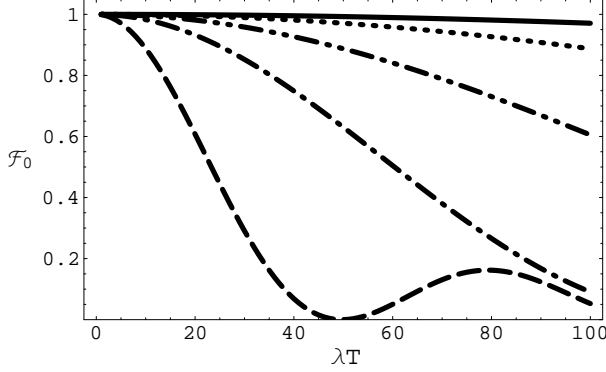
where the first two terms in the superposition are one-magnon states with flipped spin, respectively, at site  $d = 20$  and  $d = 72$ , and the last term is the superposition of two one-magnon states with the excitation placed, respectively, on site  $d = 0$  and site  $d = 5$ . The initial state is thus chosen to be a superposition of states belonging to different subspaces of Hilbert space. In the present case, the subspace of single excitations and the subspace of states of the form Eq. (8). As already discussed, such a generic initial state achieves perfect periodic reconstruction in the case of an integer value of the ratio  $B/\lambda$ . In the case of noninteger ratios  $B/\lambda$ , the state undergoes quasi-perfect periodic revivals that after few cycles begin to deteriorate. Adding more terms to the initial superposition greatly increases the number of cycles with quasi-perfect reconstruction. For this kind of initial states the analysis cannot be performed analytically in all details, as previously done for states belonging to the same subspace. However, numerics can be easily implemented for spin rings of finite, but large size. In the instance considered, we solve numerically the unitary evolution of  $\mathcal{F}_0(t)$  for a closed chain of 90 sites. Considering larger rings yields qualitatively identical results.

### 3. Other sources of noise

Concerning the issue of practical implementations, it is crucial to verify that the storing scheme does not depend critically on a perfect realization of the step-phase modulation. To analyze this dependence, in Fig. (8) we show the evolution of the fidelity  $\mathcal{F}_0(t) = |\langle\Psi_0|\psi(t)\rangle|^2$  for the same initial one-magnon state  $|\Psi_0\rangle$  previously studied, as a function of the number of periods, when the step-periodic, time-modulated phase  $\theta(t)$  Eq. (7) is approximated by its Fourier decompositions, truncated at various finite orders. Remarkably, even when considering only the first 100 harmonics, the fidelity remains close to the ideal limit  $\mathcal{F}_0(t) = 1$  for very long times. In Fig. (9) we compare this behaviour with that of a superpositional initial state. Remarkably, the fidelity of initial superpositions, although wildly oscillatory, is always larger than that of non-superpositional initial states. These results prove the stability of the finite-harmonic approximation and, as a consequence, the robustness of the

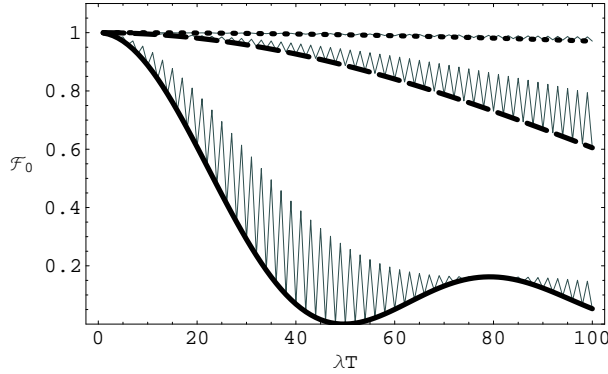


**Figure 7.** Step-periodic time-modulation of the phase Eq. (7): One-dimensional fidelity  $\mathcal{F}_0(t)$  at fixed lattice site  $d = 0$  as a function of the dimensionless time  $\lambda t$  ( $x$ -axis) in a ring of 90 qubits for the initial state  $|\chi\rangle = -(\sqrt{2}/3)|\Psi_{20}\rangle + (\sqrt{1}/3)|\Psi_{72}\rangle + (\sqrt{2}/\sqrt{3})|\Psi_{0,5}^{(2)}\rangle$  and for different values of the ratio  $B/\lambda$ . Plot A):  $B/\lambda = 2$ ,  $BT = 4\pi$ . Plot B):  $B/\lambda = 1.9$ ,  $BT = 3.8\pi$ . Notice the perfect periodic state reconstruction in plot A) (integer value of the ratio  $B/\lambda$ ), while a small deviation from it causes a very slow but progressive corruption of the quantum state revivals, as shown in plot B). In both cases the period is fixed at the value  $\lambda T = 2\pi$ .



**Figure 8.** Fidelity  $\mathcal{F}_0(t)$  on site  $i = 0$  as a function of the number of periods  $\lambda T$  for the initial one-magnon state  $|\Psi_0\rangle$  when the step-periodic time-modulated phase  $\theta(t)$  is replaced with its finite-harmonic Fourier approximations, in increasing order. Dashed line: first 5 harmonics; dot-dashed line: first 13; dot-dot-dashed line: first 25; dotted line: first 50; solid line: first 100 harmonics.

storing protocol against imperfections in the external control of the phase. Another point that needs to be addressed is the effect on the step-phase control of static, site-dependent imperfections in the magnetic field  $B$  that enters in the Hamiltonian  $\hat{H}_0$ . Remarkably, it turns out that the global topological control on the off-diagonal Hamiltonian terms allows partial control on the diagonal terms as well. Comparing the time evolutions with and without step-phase modulation, in the presence of imperfections  $\delta_i$  in  $B$ , with Gaussian distribution of not too broad half-width  $\sigma_\delta$ , one finds that in the latter case quantum diffusion grows indefinitely (as expected), while in the former case it remains limited and the fidelity undergoes revivals that can reach unity in specific cases. Moreover, the effects of the  $XY$  residual interactions continue to be completely suppressed [23].



**Figure 9.** Fidelity  $\mathcal{F}_0(t)$  on site  $i = 0$  as a function of the number of periods  $\lambda T$  for the initial one-magnon state  $|\Psi_0\rangle$  (black curves) versus the fidelity of the initial state  $|\chi\rangle = (|\Psi_1\rangle + |\Psi_0\rangle)/\sqrt{2}$  (grey curves), when the step-periodic time-modulated phase  $\theta(t)$  is replaced with its finite-harmonic Fourier approximations, in increasing order. Solid line: first 5 harmonics; dashed line: first 25; dotted line: first 100 harmonics.

#### 4. Conclusions and outlook

In Conclusion, we have introduced a scheme for the storage of information in a quantum register by a global, topological quantum control that realizes periodic, perfect state reconstruction in periodic qubit chains (rings). This kind of quantum register controlled by step-phase modulations is able to fully cancel the effects of environmental (residual) interactions of the  $XY$  type, even in the presence of local, static imperfections in the inter-qubit couplings. Moreover, the scheme is robust even in the presence of other sources of noise, such as phase modulations of finite precision and local static noise on the computational Hamiltonian [23]. The study of the effects of dynamic imperfections is under way [24]. However, it is already clear at this stage that, at least either in the ultrafast or in the adiabatic limit, the present storage scheme remains unaffected due to its global, topological nature.

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